QUANTUM MECHANICS I

1. Let E_o and E_1 be the first two energy levels of a quantum mechanical system. Let ψ_o be the wave function of the ground state. We now perform a variational calculation for the ground state and find the approximate energy level and wave function E_v and ψ_v. ψ_o and ψ_v are normalized to 1. Show that

$$\frac{E_v - E_o}{1 - |\langle \psi_o | \psi_v \rangle|^2} \geq E_1 - E_o$$

2. A spin $\frac{1}{2}$ particle with gyromagnetic ratio g and mass m is initially polarized in the plus z direction. A magnetic field of strength B is turned on at time $t = 0$ in the x direction. What is the probability of finding the polarization in the plus z direction at a time t later?

3. Consider scattering in the extreme low energy limit from a very thin, attractive, spherical shell potential

$$V(r) = -V_o \text{ for } a < r < a + \Delta a$$
$$= 0 \text{ elsewhere}$$

Find the scattering amplitude and total cross section. You may use the Born approximation and the limit $ka << 1$; indicate the restriction on the thickness Δa for this approximation to be valid.

4. a) Consider a charged particle bound in an isotropic 3-dimensional harmonic oscillator potential. Without necessarily deriving the results, what are the energy levels and their degeneracies?

4\. b) Suppose a uniform external electric field is imposed on the system in (a). How are the energy levels and degeneracies affected?

c) Suppose a uniform external magnetic field is imposed on the system in (a). Indicate how the energy levels and degeneracies are affected to lowest non-vanishing order in the field strength.

d) Is parity a good quantum number for the system described in (a)? (b)? (c)?

5\. Consider the elastic scattering of two spinless particles in an angular momentum channel ℓ, supposing that competing inelastic reactions also occur in the same channel. The partial wave elastic scattering amplitude is written

$$f_\ell = \frac{1}{k} e^{i\delta_\ell} \sin\delta_\ell \text{ where k is the wave number}$$

a) Show that the phase shift δ_ℓ must necessarily be complex.

b) What must be the algebraic sign of the imaginary part of the phase shift?

c) Derive an expression for the elastic scattering cross section in the ℓ channel.

d) Derive an expression for the inelastic scattering cross section in the ℓ channel.

e) Derive an expression relating the total cross section in the ℓ channel and the imaginary part of the partial wave amplitude.

f) What is the maximum possible value for the ratio of the inelastic to the elastic scattering cross section?

Physical Qualifying Examination UCI September 22, 1975

THERMODYNAMICS AND STATISTICAL MECHANICS

Do four problems

1. Suppose we release a cloud of cigar smoke at Irvine. Assume the smoke particles behave like other molecules in the air. Further, take our atmosphere to be at constant temperature everywhere and let there be no winds (a stagnant atmosphere; no convection). How much time (in seconds) will pass before many smoke particles (say, 10%) have spread halfway around the earth? Make a good numerical estimate.

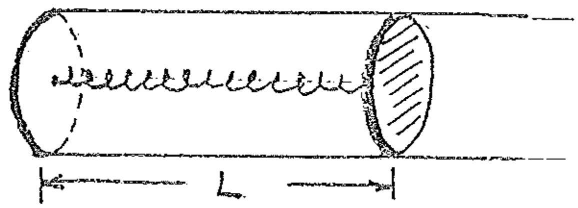
2. Consider a container of total volume V separated into two equal parts by an impermeable partition. Side one is filled with N noninteracting spin $\frac{1}{2}$ particles of mass m and side two is filled with N noninteracting spin $\frac{1}{2}$ particles of zero mass. (For instance neutrons and neutrinos respectively.) Assume that the walls are impermeable to both kinds of particles and that temperature is zero.

   a) Calculate the ratio of total energies in the two sides. (Hint: For the zero mass particles, the single particle energy $e = pc = \hbar ck$ where p = momentum and k = wave vector. For the massive particles, $\epsilon = \hbar^2 k^2/2m$. Use Fermi-Dirac statistics on these degenerate ideal gasses.)

   b) If the partition is now released, and becomes free to move (but remains impermeable to both energy and particles), calculate the ratio of equilibrum volumes.

3. Consider a metal with free electrons in it. Calculate the speed of sound in this metal. The electrons are degenerate with Fermi temperature T_F, and density n. How does this differ in basic physical ways from the sound speed in, say, a rock?

4. A gas of N noninteracting classical particles is contained inside a pipe of cross sectional area A. One end of the pipe is closed while the other is fitted with a movable piston attached by a spring, with spring constant α to the fixed end. When the piston is against the closed wall the spring is in its neutral position. The system is at temperature T. Let L be the extension of the spring.



a. What is the average value of L?

b. What is the fluctuation around this value?

5: A body obeys the equation of state

$$pV^{\alpha} = dT^{\beta} \qquad \text{(d constant)}$$

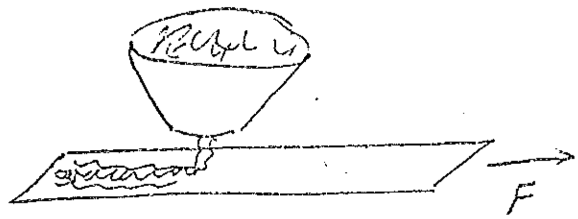
At a fixed volume V_o the specific heat is found to be independent of temperature and equal to c_v. Express the internal energy U and entropy S as a function of T and V.

Physics Qualifying Exam UCI September 24, 1975

GENERAL PHYSICS

No text. Answer 10.

1.



Sand drops from a stationary container at a rate $\frac{dm}{dt}$ (m is the mass of sand) onto a belt moving with a constant velocity v.

a. What force F is required to keep the belt moving at speed v?

b. Show that the power supplied by the force F is twice the rate of increase of kinetic energy.

c. Where has the remainder of the power gone?

2. Coherent vs Incoherent Radiating Systems

Consider n similar antennas emitting linearly polarized electromagnetic radiation of wavelength λ and velocity c. The antennas are located along the x axis at a separation λ from each other. An observer is located on the x axis at a great distance from the antennas. When a single antenna radiates, the observer measures and intensity (mean square electric field amplitude) equal to I.

a. If all antennas are driven in phase by the same generator of frequency $\nu = c/\lambda$ what is the total intensity measured by the observer?

(Cont'd next page)

b. If the antennas all radiate at the same frequency $\nu = c/\lambda$ but with completely random phase, what is the mean intensity measured by the observer?

3. One of the many ecological problems associated with nuclear electrical power generating plants is the excess thermal energy that must be discharged into the environment (thermal pollution). Why does a nuclear powered generating station present a more serious problem than a fossil fuel powered generating station? (Hint: the steam produced by a fossil fuel heated boiler is considerably hotter than that produced by a nuclear reactor.)

4. One of the most exciting new discoveries in recent years in low temperature physics has been a phase transition in liquid He^3 at very low temperatures (< 3mk). This new phase is presumably a superfluid more like that in a superconductor than like superfluid liquid He^4. Comment on the nature of a superfluid phase in a system composed of Bose particles (He^4) and a system composed of Fermi particles (He^3 and electrons in a superconductor).

5. What is the meaning of the term "adiabatic invariant" in classical mechanics? Apply this concept to the discussion of the damping of transverse oscillations in a relativistic particle accelerator. Assume that these oscillations can be described by the motion of a particle in a parabolic potential and that the damping arises from the relativistic increase of mass taking place during the acceleration. Use English sparingly; stress mathematics.

6. When you see a piece of metal, you can generally tell that it is a metal. Discuss why you can tell a metal from other forms of solids. Are there non-metals which can look like a metal?

7. Consider the elastic scattering process

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$$

for an incident anti neutrino on an electron. Derive the equation relating initial and final neutrino energies and the scattering angle.

8. A. In a glass of water at 0°C, there floats a cube of ice. Enough heat is added to melt the ice at 0°C. Does the water level rise, fall or remain constant? Why? The water temperature is raised to 2°C. Does the water level rise, fall or remain constant?

9. How do you explain the fact that a small child can "pump up" a swing without direct contact with the ground?

10. During recent weeks, several miniature rockets have been launched in the campus central park. These rockets have small engines that provide typically 10 Newton-seconds of impulse. With such an engine, what is the approximate maximum altitude of a 50 gm vehicle? What additional information would you need before you could make a more accurate calculation of the maximum altitude?

11. In Physical Review Letters last month appeared an article claiming to have experimentally detected a magnetic monopole. What properties or characteristics are conventionally assumed of the monopole? What makes it easy to detect, in principle?

12. Semiconductor devices such as transitors, diodes, etc., fabricated on silicon surfaces, are being made smaller and smaller every day. (The smallest today are $10^{-6} \times 10^{-6}$m). What physical limitations do you expect to determine the ultimate minimum size?

13. We are surrounded with solid state devices that have come from physics laboratories. Explain briefly the physical processes involved in

    magnetic core memory of computers

    light emitting diodes (LED)

    metal-oxide-semiconductor field effect transistors (MOSFET)

    Gunn oscillator

    charge coupled memory for computers

    Superconducting Quantum Interference Devices (SQUID)

Physics Qualifying Exam UCI September 22, 1975

CLASSICAL MECHANICS

Text: Goldstein or Landau.

Everyone do the first problem, then any 3 others.

1. A uniform universe of matter, initially homogeneous with density ρ_o, pressure P_o, fluid velocity $V_o = 0$, is subject to self-gravitation, described by

$$\vec{\nabla} \times \vec{g} = 0$$

$$\vec{\nabla} \cdot \vec{g} = -4\pi G\rho$$

where $\vec{g}$ is the gravitational field. The mass fluid obeys

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho\vec{v}) = 0$$

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla}p}{\rho} + \vec{g}$$

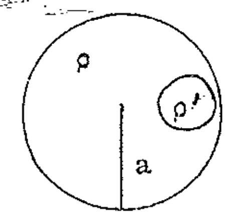
and $C_s^2 \equiv$ sound speed $\equiv \frac{\partial p}{\partial\rho}$

a. Find the "dispersion relation" for this initially homogeneous universe.

b. Some density perturbations of wave length $\frac{2\pi}{k}$ are stable, some not. Which region of k-space is stable? What is the critical k required for instability?

c. Suppose the mass fluid has some rotation at angular frequency Ω = constant. How would you expect stability to be affected? Guess the growth rate.

2. Take the earth to be a sphere of density ρ, radius a. Suppose there is a hollow sphere of radius R located somewhere inside the earth, filled with matter of density ρ'. Find



a. the acceleration $\vec{g}$ due to gravity at the point on earth's surface nearest the hollow sphere.

b. $\vec{g}$ at the point on the surface <u>farthest</u> from the hollow sphere.

3. Consider a particle of mass m in the laboratory system moving parallel to the x-axis and at a height Z_o above it for large negative x, where its velocity is (v_o, o, o). It interacts with a potential $U(\underset{\sim}{x})$ centered at the origin of coordinates. Assuming that $\underset{\sim}{F}(\underset{\sim}{x}) = - \nabla U(\underset{\sim}{x})$ is weak, solve the equations of motion of the particle

$$m\ddot{\underset{\sim}{x}} = \underset{\sim}{F}(\underset{\sim}{x})$$

to first order in $\underset{\sim}{F}(x)$ to obtain $m\dot{x}_1(t)$ $m\dot{x}_2(t)$, and $m\dot{x}_3(t)$. Specialize to the case of the Kepler problem for which $U(\underset{\sim}{x}) = \frac{\alpha}{|\underset{\sim}{x}|}$ and obtain the angle of deflection of the particle

$$\theta_1 \approx \frac{m\dot{x}_3(\infty)}{m\dot{x}_1(\infty)} = \frac{2\alpha}{m v_o z_o^2} .$$

Note: $\vec{x} = (x_1, x_2, x_3)$

4. A uniform rod slides with its ends on a smooth vertical circle of radius a. If the rod subtends an angle of $2\theta < 180^\circ$ at the center of the circle, find the frequency of small oscillations of the rod.

5. In search of more living space, an intelligent race plans to redistribute their planet's mass into a giant ring, centered on their star and spinning with angular frequency Ω. Their leading physicist argues that this design is stable. Evaluate this claim. (Drawing pictures is usually a clearer method of explanation.)

Physics Qualifying Exam UCI September 23, 1975

MATHEMATICAL PHYSICS

Text: Arfken or equivalent.

Do 4 problems.

1. Consider a perfectly flexible homogeneous string of linear mass density ς and length L, rotating with constant angular velocity Ω around a vertical axis through one end of the string.

   a. Neglecting gravity, use Hamilton's principle to show that the equation describing small transverse vibrations of the string (in a plane through the axis of rotation, rotating together with the string) is

$$\frac{\partial}{\partial x}\left((L^2 - x^2)\frac{\partial u}{\partial x}\right) - \frac{2}{\Omega^2}\frac{\partial^2 u}{\partial t^2} = 0 \qquad (1)$$

   where $u(x,t)$ denotes the deviation from the unperturbed position at distance x from the axis of rotation and time t. Show that the appropriate boundary conditions are

$$u(0,t) = 0$$
$$u(l,t) = \text{finite} \qquad (2)$$

   b. Use the method of separation of variables to find the normal modes of the string.

(continued)

c. Find the solution corresponding to the following initial conditions

$$u(x,o) = \alpha \ {}^{3} \cdot o \le x \le L \ .$$

$$u_t(x,o) = o$$

2. Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)\ \cosh\left(\frac{\pi x}{2}\right)}$$

3. A falling object in a resisting medium obeys the equation

$$m \frac{d^2x}{dt} = mg - A \frac{dx}{dt}$$

Given that $x(o) = o$ and $\frac{dx}{dt}\big|_{t=o} = o$, find $x(t)$ and $\frac{dx}{dt}$.

4. Consider the series

$$1 - n(n+1) \frac{x^2}{2!} + n(n-2)(n+1)(n+3) \frac{x^4}{4!} - \quad .$$

For what values of x does this series converge?

5. Find to two significant figures a root of

$$f = x^3 + x - 1 = 0$$

for x between 0.5 and 1.0.

6. Consider the equation

$$\frac{dy}{dx} = e^{y/x}$$

and quickly sketch a few properties of the solution in x-y space. Then suppose $y(1) = 0$. Find a series expansion in $u = x - 1$, valid near $x = 1$. Find the series to order u^3.

Physics Qualifying Exam . Sept. 24, 1975

ELECTRICITY AND MAGNETISM

Do 4 problems
Text: Jackson

1. In general, the electrical conductivity of a pure metal is a tensor $\vec{\sigma}$ defined by

$$\vec{j} = \vec{\sigma}\,\vec{E} = ne\,\vec{v}_D$$

where $\vec{j}$ is the current density, $\vec{E}$ the electric field, and $\vec{v}_D$ the average drift velocity. Calculate the electrical conductivity of a free electron gas in the present of a magnetic field $B\hat{z}$ in the z direction. Assume that the effect of collisions can be described by an energy independent collision time τ. Use classical theory.

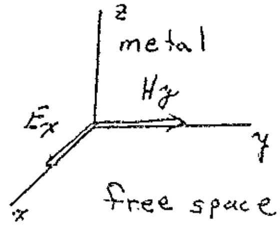
2. A square of copper is released near the earth's surface. A magnetic field

$$B = B_o \left(\frac{A}{r}\right)^3 , \quad \text{exist for } r > R$$

where R is the earth's radius, r the distance from the earth's center and A a constant. Assume the square is released at some radius R' and during its fall $\vec{B}$ is parallel to the earth's surface and normal to the plane of the square at all times. The square has side length L, thickness d, constant resistivity ρ, and mass m. Gravitational acceleration g is constant. Find the square's vertical velocity v as a function of time.

3. The surface impedance of a semiinfinite metal filling the half space $z > 0$ is defined as

$$Z = \frac{E_x(z=0)}{\int_0^\infty J_x(z)\,dz} .$$



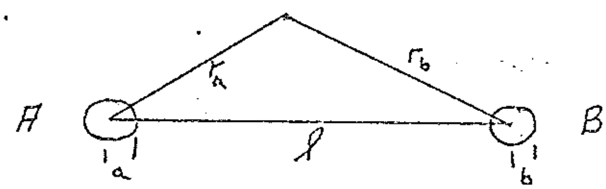
Show from Maxwell's equations that Z can also be written as

$$Z = \frac{E_x(z=0)}{H_y(z=0)} = i\,\mu_o\,\omega\,\frac{E_x(z=0)}{\left.\frac{\partial E_x}{\partial z}\right|_{z=0}}$$

in S.I. units. (Hint: assume $\vec{E} = \hat{i}\,E_x e^{i(kz-\omega t)}$;

$\vec{H} = \hat{j}\,Hy\,e^{i(kz-\omega t)}$ and neglect the displacement current.)

4. Two small spherical electrodes A and B of radius a and b are located a distance ℓ apart in an infinite conducting medium of conductivity σ. Find the resistance between the two electrodes.

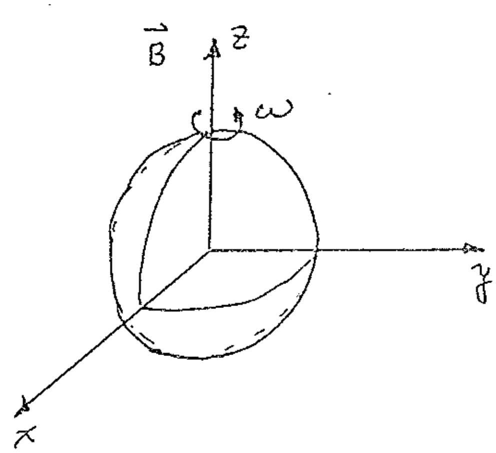


(Hint: First find the potential V at an arbitrary point r_a from A and r_b from B for a current I flowing between the two electrodes by using the principle of superposition. Then find $V_a - V_b$.)

5. Find the electric potential everywhere produced by a dielectric sphere rotating with angular frequency ω about the direction of a uniform magnetic field. Assume that the polarization $\vec{P}$ is given by

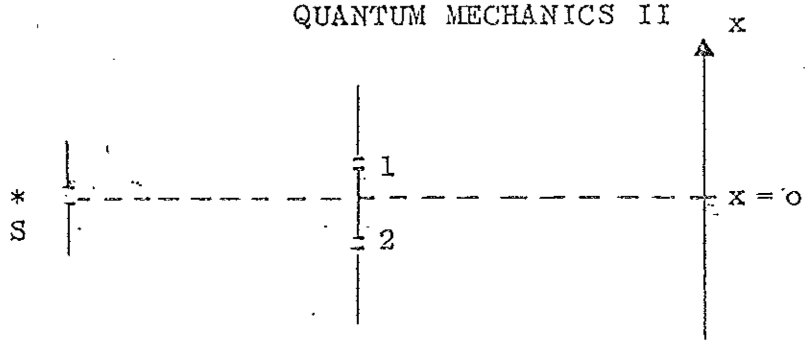
$$\vec{P} = (\epsilon - 1)\{\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\}$$

where ϵ is the dielectric constant of the sphere and v is velocity of the dielectric. Assume that the conductivity is zero and the total charge on the sphere is zero.



QUANTUM MECHANICS II

1.



Suppose a strong source, S, emits a high flux of monoenergetic electrons which pass through slits 1 and 2, and fall on a detector capable of measuring the position x of the electrons.

Sketch, as a function of x, the rate at which electrons are detected on the screen under the following circumstances

a) Slit 1 open, 2 closed

b) Slit 2 open, 1 closed

c) Slits 1 and 2 both open.

In what way would the patterns in (a), (b), (c) differ if the source were very weak, i.e., if there were a negligible probability of two or more electrons arriving simultaneously at the screen within the resolving time of the detector?

2. The Hamiltonian for a free Dirac particle is

$$H = c\,\vec{\alpha}\cdot\vec{p} + \beta m c^2$$

a) What is the velocity operator?

b) What are its eigenvalues?

c) Discuss the significance of the result (b).

3. The 21 cm line used by the radio astronomers is a magnetic dipole transition between the hyperfine levels of the ground state of atomic hydrogen.

a) What are the various quantum numbers of these energy levels?

b) Explain why the transition is magnetic dipole.

c) Discuss the effect of a uniform <u>weak</u> magnetic field on the spectral line (i.e. the weak field Zeeman effect).